CHAPTER TIME VALUE OF MONEY (TOPPERS INSTITUTE KANPUR)

4.1 BASICS

A. Concept of Interest: Interest is the remuneration paid by the borrowers of money to the lenders. The excess money paid to the Lender of Funds is called Interest. **Example:** If Rs. 1,00,000 is borrowed for a year, the amount of repayment would be in excess of the amount so borrowed. If say Rs. 1,10,000 is paid back Rs. 10,000 constitutes interest. The reasons behind the payment of Interest are as follows -

1. **Time Value of Money:** Time Value of Money means that the value of a unit of money is different in different time periods. The sum of money received in future is less valuable than it is today. In other words, the present worth of Rupees received after some time will be less than a Rupee received today. Since a Rupee received today has more value, rational investors would prefer Current Receipts to Future Receipts. If they postpone their receipts, they charge some interest.

2. Opportunity Cost: The Lender has a choice between using his money in different investments. If the chooses one he forgoes the return from all others. In other words, the Lender incurs an Opportunity Cost due to the possible alternative uses of the lent money.

3. **Inflation:** Most economies generally exhibit inflation. Inflation is a fall in the purchasing power of money. Due to inflation, a given amount of money buys fewer goods in the future than it will now. The Borrower needs to compensate the Lender for this.

4. Liquidity Preference: People prefer to have their resources available in a form that can immediately be converted into cash rather than a form that takes time for money to realize.

5. Risk Factor: There is always a risk that the Borrower will go bankrupt or otherwise default on the Loan. Risk is a determinable factor in fixing rate of interest. A Lender generally charges more interest rate (Risk Premium) for taking more risk.

B. Other Terms Involved:

1. **Principal:** Principal is initial value of Lending or Borrowing. In case of Investment, the value of Initial Investment , is also called Principal.

2. Rate of Interest: The rate at which the interest is charged for a defined length of time for use of principal generally on a periodical basis is known to be the Rate of Interest. Rate of Interest is usually expressed as a percentage. **Example:** If the Interest Rate is 5% per annum, it means that Rs. 5 would be paid as interest, for every Rs. 100 of principal amount in a year.

3. **Accumulated Amount (Balance):** Accumulated Amount is the final value of an investment. It is the sum total of Principal and Interest earned. **Example:** If Rs. 1,00,000 is borrowed for a year at 10% p.a, the amount of interest earned for a year would be Rs. 10,000. At the end of the period, Rs. 1,10,000 would remain on Account. The same is also known as Amount or the balance due.

C. Methods of Analysis: The concept of Time Value of Money helps in arriving at the comparable value of the different rupee amount arising at different points of time into equivalent values of a particular point of time (Present or Future). This can be done by either -

(a) **Compounding** the Present Money to a future date, i.e. finding out **Future Value of Present Money,** or

(b) **Discounting** Future Money to the present date, i.e. finding out **Present Value of Future Money.**

Present Date	Compounding →	Future Date
Present Cash Flows or Present Money	← Discounting	Future Cash Flows or Future Money

4.2 SIMPLE INTEREST

Simple Interest is the interest calculated as a simple percentage of the original principal amount. The formula relating to Simple Interest are given below -

Simple Interest = **P × N × R,** where P = Principal Amount. N = Number of years. R = Interest Rate per annum.	**Amount = Principal** + **Interest** Hence, A = P + (P × N × R), **A** = **P [1 + (N × R)]**

Example:

Let us say that Rs. 50,000 is invested at Simple Interest of 10% p.a. for 3 years. The amount at the end of 3 years as per the formula = 50,000 [1+ (0.10 × 3)] = 50,000 × 1.3 = Rs. 65,000.

Year	Opening Principal	Interest	Total Interest	Amount
1	50,000	=50,000×10%=5,000	5,000	55,000
2	50,000	=50,000×10%=5,000	10,000	60,000
3	50,000	=50,000×10%=5,000	15,000	65,000

4.3 COMPOUND INTEREST

1. Compounding means that interest is paid both on previous earned interest and as well as on the Principal.

2. Interest due at the end of every payment period is added to the Principal, and Interest on the next payment period is computed on the new Principal.

3. The Time Interval between successive additions of interests is known as Conversion (or Payment) Period. Some Conversion Periods are -

Conversion Period	1 day	1 month	3 months	6 months	12 months
i.e. Compounded	Daily	Monthly	Quarterly	Half-yearly	Annually

4. The amount (Principal + Interest) at the end of every payment period is indicated below -

Payment Period	First	Second	Third	n^{th}
Amount Due=A=	$A_1 = P + PR$ $= P(1 + R)$	$A2 = A_1 + A_1 \times R$ $= A_1(1 + R)$	$A_3 = A_2 + A_2 \times R$ $= A_2(1 + R)$	$An = A_{n-1} + A_{n-1} \times R$ $= A_{n-1}(1 + R)$

		$= P(1 + R)^2$	$= P(1 + R)^3$	$= P(1 + R)^n$

Note: Amount under Compound Interest = $\mathbf{P(1 + R)^{NK}}$

Where

P = Principal Amount.

N = Number of years.

K = Number of times compounding is done per year, e.g. Monthly (12), Quarterly (4), etc.

R = Interest Rate per payment period = $\frac{\text{Interest Rate p.a.}}{\text{Number of payment periods p.a.}} = \frac{I}{K}$

Additional Points:

(a) **Compound Interest Tables:** Compound Interest Tables as well as Future Value (FV) Tables for $\mathbf{(1 + R)^{NK}}$ at various rates per annum with - (a) annual, (b) monthly, and (c) daily compounding, are available for easy calculation.

(b) **Rule of 72:** In case of Compound Interest, the number 72 provides an interesting approximation. If we divide 72 by the Interest Rate, we can estimate the number of years it takes for the money to double (for or against) us. For example, if interest rate is 12%, it takes (72÷12 =) 6 years approximately for the money to become **double.**

(c) **Simple vs Compound Interest:** The longer the funds are invested, the greater the advantage with Compound Interest than with Simple Interest. The rate of growth is higher in terms of Compound Interest.

Example: Let us say that Rs. 50,000 is invested at Compound Interest of 10% p.a. for 3 years.

Year	Opening Principal	Interest	Amount
1	50,000	=50,000 × 10% = 5,000	55,000
2	55,000	=55,000 × 10% = 5,500	60,500
3	60,500	=60,500 × 10% = 6,050	66,550

1. Amount at the end of n years of the amount P invested at Time 0 at the rate of R is = $P(1 + R)^n$

In the above example, Amount at the end of the Third Year = $50,000(1+0.10)^3$ = Rs. **66,550.**

2. Total Interest for the period = Amount at the end of n years - Opening Principal = $P(1 + R)^n - P = P[(1 + R)^n - 1]$

In the above example, Compound Interest for 3 year period = 66,550 - 50,000 = Rs. **16,550.**

4.4 NOMINAL HATE AND EFFECTIVE INTEREST RATE

1. **Nominal Rate:** The Annual Compound Interest Rate is called the **Nominal Interest Rate.**

2. **Effective Rate:** When the amount is compounded more than once a year, the actual rate of interest for each year is called the Effective Interest Rate. Effective Rate arises only when there is more than one compounding terms per

annum. Effective Interest Rate is computed as $E = (1 + i)^m - 1$, where m is number of compounding terms per annum. Where Compounding is done more than once during an annum, the Effective Rate > Nominal Rate.

3. **Formula:**

(a) **Effective Interest RATE=** $\frac{\text{Actual Interest Paid During the Year}}{\text{Opening Principal of the Year}} = \frac{\text{Closing Amount} - \text{Opening Principal}}{\text{Opening Principal of the Year}}$

(b) If interest is paid "k" times in a year, & "i" is the rate of interest per annum, **Effective Rate of Interest** (E) is

given by E =$\left(1+\frac{i}{k}\right)k-1$

Note: The Concept of Effective Interest Rate arises only where there is more than one compounding terms per annum.

4.4 CONCEPT OF ANNUITY

1. Meaning: An **Annuity** is a stream or sequence of regular periodic Cash Flows (either payments made or received) for a specified period of time. Some examples of Annuity Payments are - (a) Recurring Deposit Instalments paid to a Bank, (b) Life Insurance Premium per annum, (c) Sinking Fund Instalments, etc.

2. **Features:**

(a) Annuity refers to a series of payments and not one payment.

(b) Amount paid should be constant over the period of annuity.

(c) Time interval between two consecutive Payments (or Receipts) should be the same.

Year end	**Payments/ Receipts(Rs.)**	**Payments/Receipts(Rs.)**	**Payments/ Receipts(Rs.)**
1	5000	5000	5000
2	6000	5000	5000
3	4000	-	5000
4	5000	5000	5000
5	7000	5000	5000
Annuity?	Not an annuity, since Cash Flows are **not equal**	Not an annuity, since Cash Flows are **not regular.**	This is an Annuity.

3. **Annuity Regular vs Annuity Immediate:**

Annuity Regular	**Annuity Immediate**
Regular Payments or Receipts are made at the **end** of each year / period.	Payments or Receipts is made at the **beginning** of each year / period.
First Payment or Receipt arises at the end of **Year 1.**	First Payment or Receipt arises immediately, i.e. in **Year 0.**

4.5 CONCEPT OF COMPOUNDING AND FUTURE VALUE

1. **Compounding** the present money to a future date involves finding out **future value of present money.**

2. **Future Value: Meaning:** Future Value represents value at the end of n^{th} year. Future Value is the cash value of an investment at some time in the future. It is tomorrow's value of today's money compounded at the rate of interest. **Example:** If Rs. 1000 is invested in a

Fixed Deposit that pays you 7% per annum as interest. At the end of the year the amount would be Rs. 1070. This consists of the Original Principal of Rs. 1000 and the interest earned of Rs. 70. Rs. 1070 is the Future Value of Rs. 1000 invested for one year at 7%. Similarly, it can also be concluded that Rs. 1000 today is worth Rs. 1070 in one year's time if the interest rate is 7%.

3. **Future Value Formula:**

A. **Future Value of a Single Cash Flow:** We know that, $A = P(1+i)^n$. So, the Future Value of a **Single Cash Flow** represented as FV, is given by the Formula, **FV** = **CF $(1+i)^n$;** where CF = Cash Flow.

B. **Future Value of an Annuity Regular:** It is the value of the Annuity at the end of n years.

Example: Consider an Annuity Payment of Rs. 5,000 for 5 years. The Value at the end of the 5^{th} year would be Rs. 30,526 at Interest of 10% p.a compounded annually. This can be calculated as shown below -

Year end	Payments/Receipts(Rs.)	Future Value
1	5000	$= 5000 \times (1.1)^4 = 7321$
2	5000	$= 5000 \times (1.1)^3 = 6655$
3	5000	$= 5000 \times (1.1)^2 = 6050$
4	5000	$= 5000 \times (1.1)^1 = 5500$
5	5000	$= 5000 \times (1.1)^0 = 5000$
	Future Value of Annuity Cash Flow	**Rs. 30,526**

For Annuity Regular, Future Value of an Annuity=CF × $\frac{[(1+R)n-1]}{R}$	Where P = Cash Flow in each period, n = number of years for which the money is invested, R = Rate of Return on the Investment.

Computation: For the same example as above, the Future Value = $\frac{5000}{0.10}[(1+0.1)^5 - 1]$ = Rs. 30,526

C. **Future Value of an Annuity Immediate:**

Example: Consider an Annuity Payment (as an Annuity Immediate) of Rs. 5,000 for 5 years. The Value at the end of the 5^{th} year at Interest of 10% p.a compounded annually can be calculated as shown below -

Year end	Payments/Receipts(Rs.)	Future Value
0	5000	$= 5000 \times (1.1)^5 = 8053$
1	5000	$= 5000 \times (1.1)^4 = 7321$
2	5000	$= 5000 \times (1.1)^3 = 6655$

3	5000	= 5000 × $(1.1)^2$ = 6050
4	5000	= 5000 × $(1.1)^1$ = 5500
	Future Value of Annuity Cash Flow	**Rs. 33,579**

Future Value of Annuity Immediate = Future Value of Regular Annuity × (1 + R)

i.e. Future Value of an Annuity Immediate = CF × $\frac{[(1+R)n-1]}{R}$ × (1+R)

Computation: For the same example as above, the Future Value = $\frac{5000}{0.10}$ [$(1 + 0.1)^5$ -1] × (1+0.1) = Rs. **33,579**

4.6 CONCEPT OF DISCOUNTING AND PRESENT VALUE

1. Present Value: Present Value (PV), is the amount of money that represents the sum of Principal and Interest, if such amount (say Rs. P) is required to be invested now at a certain rate compounded over a number of time periods at a specified rate for each time period.

Present Value is the Value today of the money to be received at a future point of time. In the above example Rs. 1070 was to be received at the end of Year 1 after considering interest at 7% p.a. Therefore the Present Value or the Value now of the Rs. 1,070 to be received after 1 year is Rs. 1,000.

From the formula of Amount under Compound Interest, we know that $A_n= P(1 + R)^n$

Transposing the above formula & solving for P, we have P = $A_n (1 + R)^{-n}$. So, Present Value= $\frac{\mathbf{A_n}}{\mathbf{(1+R)^n}}$

2. **Discounting** future money to the present date involves finding out Present Value of future money. Discounting is the opposite of Compounding and hence, Present Value of Future Cash Flows is given by the following formula –

$$\text{Present Value} = \frac{FV_1}{(1+R)^1} + \frac{FV_2}{(1+R)^2} + \frac{FV_3}{(1+R)^3} + \frac{FV_4}{(1+R)^4} + \cdots \ldots \ldots \ldots .. \frac{FV_n}{(1+R)^n}$$

Where l,2,3,4,....n represent future time periods and FV = Cash Flows arising at those future points of time, and R denotes the Discount Rate / Rate of Interest.

3. **Present Value Formula:**

A. **Present Value of a Single Cash Flow:** Present Value of a Single Cash Flow is given by the Formula PV = $\frac{CF}{(1+R)^n}$.

B. **Present Value of an Annuity Regular:** PV of an Annuity means the Value of the Annuity at present. It represents the expected Future Cash Flows, at a given rate of interest. Consider the Annuity as under -

Year end	Payments/Receipts(Rs.)	Present Value
1	5000	= 5000 ÷ $(1.1)^1$ = 4545

2	5000	$= 5000 \div (1.1)^2 = 4132$
3	5000	$= 5000 \div (1.1)^3 = 3757$
4	5000	$= 5000 \div (1.1)^4 = 3415$
5	5000	$= 5000 \div (1.1)^5 = 3105$
	Present Value of Annuity Cash Flow	**Rs. 18,954**

Present Value of an Annuity Regular = CF × $\frac{[(1+R)_n - 1]}{R\,(1+R)^n}$ (i.e. in other words PV = $\frac{\text{FV of Annuity Regular}}{(1+R)^n}$

Where n = number of years for which the money is invested, R = rate of return on the investment.

Computation: In the example in Page 4.4, the Future Value of the Rs. 5,000 annuity for 5 years at 10% p.a.

compounded annually, was Rs. 30,526. Therefore the Present Value is $\frac{30526}{1.1^5}$ = Rs. 18,954.

C. **Present Value of an Annuity Immediate:** Consider the Annuity as under -

Year end	Payments/ Receipts(Rs.)	Present Value
0	5,000	Amount arises now, so PV = 5000
1	5000	$= 5000 \div (1.1)^1 = 4545$
2	5000	$= 5000 \div (1.1)^2 = 4132$
3	5000	$= 5000 \div (1.1)^3 = 3757$
4	5000	$= 5000 \div (1.1)^4 = 3415$
	Present Value of Annuity Cash Flow	**Rs. 20,849**

PV of Annuity Immediate can be computed in two different ways -

Method	Example
(a) PV of Annuity Immediate=$\frac{\text{FV of Annuity Immediate}}{(1+R)^n}$	In the example in Page 4.4, FV of Annuity Immediate was Rs. 33,579. So, PV= $\frac{33579}{1.1^5}$ Rs. **20,849.**
(b) Compute PV for (n-1) years and then add back the Annuity Amount (relating to Year 0).	PV of Annuity 5,000 for n-1, i.e. 5-1 = 4 yrs, at 10% is $= [5{,}000 \times \frac{(1+0.1)^4-1}{0.1\times(1+0.1)^4}] + 5{,}000$

	= 15,849 **+** 5,000 **= Rs. 20,849**

4.7 CONCEPT OF PERPETUITY

1. **Meaning:** Perpetuity is a stream of payments or a type of annuity that starts payments on a fixed date and such payments continue forever, i.e. **perpetually.** Thus, Perpetuity is a constant stream of identical annual cash flows with no end, i.e. upto infinity.

2. **Examples:** (a) Dividend on Irredeemable Preference Share Capital, (b) Interest on Irredeemable Debt / Bonds, (c) Scholarships paid perpetually from an Endowment Fund, etc.

3. **Operation:** In a Fund involving perpetual annual Cash Flows, an Initial Fund (Principal) is established first, and the payments will flow from the Fund indefinitely. This means that these periodic payments are effectively the annual interest payments.

4. **Value:**

(a) Value of a Perpetuity is calculated as its Expected Income Stream -r Discount Factor or Market Rate of Interest. Thus, it reflects the expected Present Value of all payments (to be received perpetually).

(b) The Value of a Perpetuity is **finite** because receipts that are anticipated far in the future have extremely low PV. Also, because the Principal is never repaid, there is no Present Value for the Principal.

Note: Since Perpetuity is a type of annuity which is unending, its sum or Future Value **cannot** be calculated.

5. **Formula:**

PV of a **Constant** Perpetuity = $\frac{C}{R}$	Where **C** = Cash Flow, i.e. Interest, Dividend, etc. per period. **R** = Interest Rate per payment period.

PV of a **Growing** Perpetuity = $\frac{C}{R-G}$	Where **C** = Cash Flow, i.e. Interest, Dividend, etc. for the first period **R** = Interest Rate per payment period. **G** = Rate of growth in Cash Flows.

Note: A stream of Annual Cash Flows growing at a constant rate forever is known as **Growing** Perpetuity.

6. **Annuity vs Perpetuity:**

Particulars	Annuity	Perpetuity
Meaning	An **Annuity** is a stream of regular periodic cash flows (either payments made or received) for a specified period of time.	Perpetuity is a stream of payments or a type of annuity that starts payments on a fixed date and such payments continue forever, i.e. **perpetually.** Thus, Perpetuity is a constant stream of identical cash flows with no end.

Examples	(a) Recurring Deposit Instalments paid to a Bank, (b) Life Insurance Premium per annum.	(a) Dividend on Irredeemable Preference Share Capital, (b) Interest on Irredeemable Debt / Bonds, (c) Scholarships paid perpetually from an Endowment Fund, etc.
FV	Future Value of Annuity can be computed using Compounding Technique.	Perpetuity is a type of annuity which is unending, its sum or Future Value **cannot** be calculated.

4.8 APPLICATIONS

1. Sinking Fund: It is a fund created for a purpose by way of sequence of periodic payments over a period of time at a specified rate. Interest is compounded at the end of every period. The Sinking Fund Payments generally take the form of an Annuity.

The Future Value of the Sinking Fund Annuity $= \mathrm{A(n,i)} = \mathrm{A}\left[\frac{(1+\mathrm{i})^{\mathrm{n}}-1}{\mathrm{i}}\right]$ (Note: This is the same as FV of Annuity)

2. **Leasing:** Leasing is a financial arrangement under which the owner of the asset (Lessee) allows the User of the asset (Lessee) to use the asset for a defined period of time (lease period) for a consideration (Lease Rental) payable over a period of time. The Lease Rentals form an **annuity** as they are fixed payments made at regular intervals. The Present Value of the Lease Rental Annuity (using Formula given above) represents the Cash Down Price of the Lease to the Lessee and Present Value of Lease Income to the Lessor.

3. **Capital Expenditure (Investment Decision):**

(a) Capital Expenditure means purchasing an asset today (which results in outflow of money at Time 0) in anticipation of benefits (cash inflow) which would flow across the life of the Investment.

(b) The **Present Value** of Cash Outflows and Inflows (using formula given above) are compared to arrive at the

Investment decision.

Particulars	Investment Decision
Present Value of Cash Inflow > Present Value of Cash Outflow	Investment to be made
Present Value of Cash Inflow < Present Value of Cash Outflow	Investment not to be made
Present Value of Cash Inflow = Present Value of Cash Outflow	Indifference Point

4. **Valuation of Bond:** Bonds are long-term debt securities, which enables Purchasers to receive periodic interest payments until maturity at which they receive face value of the bond. Value of Bond = Present Value of Interest Payments + Present Value of Maturity payments, (to be found using the formula for Present Value)

5. **Compounded Annual Growth Rate (CAGR):** This is given by the following formula –

CAGR = $\sqrt[\mathrm{n}]{\frac{\text{Total Return+Initial Investment}}{\text{Initial Investment}}}$ –1[Inverse of Compound Interrest Formula]

OR = $\left[\frac{\text{Total Return+Initial Investment}}{\text{Initial Investment}}\right]^{-1/n} - 1$ where, “n” represents the period of holding.

6. **Effect of Inflation:** Since Cash-Flows are expressed in Nominal Terms (after including the impact of inflation), the discount rate chosen should also be Nominal Discount Rate, i.e. Real Discount Rate adjusted for inflation.

Hence, $(1 + R_N) = (1 + R_R) \times (1 + I)$

Where, R_N = Nominal Discount Rate and I = Inflation Rate

R_R = Real Discount Rate (Without effects of inflation)

Illustrations

4.1 SIMPLE INTEREST

Illustration 1: Simple Interest Basics

Answer the Following -

(a) How much interest will be earned on Rs. 2,000 at 6% simple interest for 2 years?

Solution: Required Interest Amount is given by I = P × i × n = 2,000 × 0.06 × 2 = Rs. 240

(b) X deposited Rs. 50,000 in a bank for two years with the

interest rate of 5.5% p.a. How much interest would she earn?

What would be the Final Value of the Deposit?

Solution:

Required Interest Amount is given by I = P × i × n

= Rs. 50,000 × 0.055 × 2 = Rs. 5,500

Final value of investment is given by A = P(1 + in)

= Rs. 50,000$(1+\frac{5.5}{100}\times 2)$=Rs. 50,000$(1+\frac{11}{100})$

= Rs. 50,000 $\times\frac{111}{100}$ = Rs. **55,500** 100

Alternatively, A = P + I = Rs. (50,000 + 5,500) = Rs. **55,500**

(c) Rahul deposited Rs. 1,00,000 in his bank for 2 years at simple interest rate of 6%. How much interest would he earn? How much would be the final value of deposit?

Solution:

(a) Required interest amount is given by I = P × i × n

= Rs. 1,00,000 × $\frac{6}{100}$ × 2 100

= Rs. 12,000

(b) Final value of deposit is given by

Amount = Principal + Interest

= Rs. (1,00,000 + 12,000)

= Rs. 1,12,000

Illustration 2: Simple Interest Solving for Missing Variables Answer the Following -

Find the rate of interest if the amount owed after 6 months is Rs. 1,050, borrowed amount being Rs. 1,000.

Solution: We know A = P +(P× i × n)

i.e. 1,050 = 1,000 + (1,000 × i × 6 /12)

50 = 500i ; i = 1/10 = 10%

Rahul invested Rs. 70,000 in a bank at the rate of 6.5% p.a. **S.I..** He received Rs. 85,925 after the end of term. Find out the period for which sum was invested by Rahul.

Solution: We know A = P (1 + in)

i.e. 85,925 = 70,000 $(1+\frac{6.5}{100}\times n)$

$$\frac{85{,}925}{70{,}000} = \frac{100+6.5n}{100}$$

$$\left(\frac{85{,}925}{70{,}000}\times 100\right) - 100 = 6.5n$$

6. $5n = 22.75$; $n = 3.5$ . . time = 3.5 years

Kapil deposited some amount in a bank for $7^1/_2$ years at therate of 6% p.a. simple interest. Kapil received Rs. 1,01,500 at theend of the term. Compute initial deposit of Kapil.

Solution: We know A = P (1 + in)

1,01,500 - P(1+$\frac{6}{100}$×$\frac{15}{2}$)

1,01,500 = P x1+ $\frac{45}{100}$

1,01,500 = P ×$\left(\frac{145}{100}\right)$

$\mathrm{P} = \frac{1,01,500 \times 100}{145}$ = Rs 70,000

∴ Initial deposit of Kapil = Rs. 70,000

A sum of Rs. 46,875 was lent out at simple interest and at the end of 1 year 8 months the total amount was Rs. 50,000. Find the rate of interest percent per annum.

Solution: We know A = P (1 + in)

i.e. 50,000 = 46,875 (1 + i × 1 $\frac{8}{12}$)

$\frac{50,000}{46,875} = 1 + \frac{5}{3}\mathrm{i}$

$(1.067 - 1) \times \frac{3}{5} = \mathrm{i}$

$\mathrm{i} = 0.04$; rate = 4%

Illustration 3: Simple Interest Solving for Missing Variables

What sum of money will produce Rs. 28,600 as an interest in 3 years and 3 months at 2.5% p.a. simple interest?

Solution:

I = P × i × n

28600 = P × $\frac{2.5}{100}$ × 3$\frac{3}{12}$

28600 = P × $\frac{2.5}{100}$ × $\frac{13}{4}$

28600 = P × $\frac{32.5}{400}$

P= × $\frac{28,600 \times 400}{32.5}$ = 352000

∴ Rs. 3,52,000 is to be invested to produce Rs. 28,600 interest in 3 years and 3 months at 2.5% p.a. simple interest

In what time will Rs. 85,000 amount to Rs. 1,57,675 at 4.5% p.a.?

Solution:

A = P (1 + in)

1,57,675 = 85,000 (1 + 0.045 × n)

$\frac{1,57,675}{85,000} = 1 + 0.045\mathrm{n}$

$0.045\mathrm{n} = \frac{1,57,675}{85,000} - 1$

$\mathrm{n} = \frac{0..855}{0045} = 19$ years

∴Rs. 85,000 will amount to Rs. 1,57,675 at 4.5% p.a. simple interest rate in 19 years.

4.2 COMPOUND INTEREST

Illustration 4: Compound Interest

Saina deposited Rs. 1,00,000 in a nationalized bank for three years. If the rate of interest is 7% p.a., calculate the interest that bank has to pay to Saina after three years if interest is compound annually. Also calculate the amount at the end of third year.

Solution:

Year	Opening Principal	Interest = P×i×t	Closing Principal
1	1,00,000	1,00,000× 0.07 × 1 = 7,000	1,07,000
2	1,07,000	1,07,000× 0.07 × 1 = 7,490	1,14,490

3	1,14,490	1,14,490×0.07× 1= 8014.30	1,22,504.30
Total		**22,504.30**	

Note: The Calculation can also be found out as follows -

Amount at the End of the Period = $P(1+i)^n$ = I,00,000$(1+07)^3$ = 1,00,000 (1.225043) = 1,22,504.30

Total Interest = Closing Principal - Opening Principal = Rs. 22,504.30

Illustration 5: Compound Interest - Multiple Compounding

Rs. 2000 is invested at annual rate of interest of 10%. What is the amount after two years if compounding is done (a) Annually (b) Semi-annually (c) Quarterly (d) Monthly.

Solution:

In case Compounding is done more than once a year the Formula to be used is as follows - **$A = P(1+I)^N$,**

Where, P = Amount Invested or Principal (P = 2000 Given)

I = i/m ;

N is the number of compounding terms in the period=mn, m is the number of Compounding in one year

<table>
<tr>
<td>(a) Annual Compounding
Since the interest is compounded yearly the number of conversion periods n in 2 years are 2, n=2. i=0.10
$A = P (1 + i)^n$
$A = Rs.\ 2000\ (1 + 0.1)^2$
$= Rs.\ 2000 \times (1.1)^2 = Rs.\ 2000 \times 1.21$= Rs. 2,420</td>
<td>(b) Semi-Annual Compounding
In case of Semi-Annual Compounding
m= no. of compounding terms in 1 year = 2
N = no. of compounding terms in 2 years = mn = 2x2 = 4
I = i/2 = 0.1/2 = 0.05
$A = 2000\ (1 + 0.05)^4$= 2000 × 1.2155 = Rs. 2,431</td>
</tr>
<tr>
<td>(c) Quarterly Compounding
In case of Quarterly Compounding
m= no. of compounding terms in 1 year = 4
N = no. of compounding terms in 2 years = mn = 2×4 = 8
I = i/4 = 0.1/4 = 0.025
$A = 2,000\ (1 + 0.025)^8$ = 2,000 × 1.2184= Rs. 2,436.80</td>
<td>(d) Monthly Compounding
In case of Monthly Compounding
m = no. of compounding terms in 1 year = 12
N = no. of compounding terms in 2 years = mn = 2 × 12=24
I = i/12 = 0.1/12 = 0.00833
$A = 2000\ (1 + 0.00833)^{24}$
= 2000 × 1.22029 = Rs. 2,440.5893</td>
</tr>
</table>

Illustration 6: Compound Interest - Multiple Compounding

Determine the compound amount and compound interest on Rs. 1000 at 6% compounded semi-annually for 6 years. Given that $(1 + i)^n = 1.42576$ for i = 3% and n = 12.

Solution:

In case of semiannual compounding

m= no. of compounding terms in 1 year = 2

N = no. of compounding terms in 2 years = mn = 2 x 6 =

12 I = i/2 = 0.06/2 = 0.03

P = 1000

Amount at the End of n years = A= P $(1 + 1)^N$ = Rs. 1000 $(1 + 0.03)^{12}$ = 1000 x 1.42576 = Rs. 1425.76

Compound Interest = Amount - Opening Principal = 1425.76 - 1000 = Rs. 425.76

Illustration 7: Compound Interest - Multiple Compounding

Compute the compound interest on Rs. 4,000 for $1^1/_2$ years at 10% per annum compounded half-yearly.

Solution:

A = P $(1 + i)^n$

P = Rs. 4,000.

m = 2 (in case of Semi - Annual Compounding)

I = i/m = 0.1 / 2 = 0.05

N = nm = 1.5 × 2 = 3

Amount at the End of 3 Compounding Terms A = 4,000 $(1 + 0.05)^3$ = 4,630.50

= 4,630.50-4,000 = Rs. 630.50

Note:

1. Alternatively, C.I. (Compound Interest for N Periods) = P[(I + $i)^n$ - 1]

2. The Compound Interest Formula connects C.I., P, I and n. Where, three out of these four variables are given the fourth can be found out by simple calculations.

Illustration 8: Solving for Missing Variables

(a) On what sum will the compound interest at 5% per annum for two years compounded annually be Rs. 1,640? Solution: n = 2 ; i = 0.05 C.I. =P[$(1 + i)^n$ - 1] 1640 = P [$(1 + 0.05)^2$ - 1] 1640 =P (1.1025- 1) P = 1,640 / 0.1025= 16,000 ∴Rs. 16,000 is to be invested to earn Interest of Rs. 1,640, in 2 years time at 5% Compounded Annually.	(b) What annual rate of interest compounded annually doubles an investment in 7 years? Given that $2^{1/7}$ = 1.104090 Solution: If the Principal be P then A = 2P. A = $P(1 + i)^n$ 2P = P $(1 + i)^7$ => 2 = $(1 + i)^7$ Taking the 7^{th} Root Both Sides , $2^{1/7}$ = (1 + i) 1.104090 = 1 + i I = 0.10409 ∴ Required Rate of Interest = 10.41% p.a.

(c) When will Rs. 8,000 amount to Rs. 8,820 at 10% per annum interest compounded half-yearly? **Solution:** m = 2, I = i/m = i/2 = .10/2 = 0.05 , N =mn = 2n (Reqd) Principal (P)= Rs. 8,000 Amount (A)= Rs. 8,820 We know A= P (1+ I)^{N} 8,820 = 8,000 (1+0.05)^{N} $\frac{8820}{8000} = (1.05)^{N}$ $1.1025 = (1.05)^{N}$ (1.05)2 $= (1.05)^{N}$ $N = 2$ $N = 2 = 2n, n = 1, \therefore$ Number of $\text{Years} = 1.$	(d) Find the rate percent per annum if Rs. 2,00,000 amount to Rs. 2,31,525 in $1^{1}/_{2}$ year interest being compounded half-yearly. **Solution:** Here P = Rs. 2,00,000 Amount (A) = Rs. 2,31,525 m = 2, N = no. of conversion periods = mn= $1^{1}/_{2} \times 2 = 3$ I = i/m = i / 2. We know that, A = P (1 + I)^{N} 2,31,525 = 2,00,000 (1 + I)^{3} $\frac{2,31,525}{2,00,000}$ 1.157625 = (1 + I)^{3} (1.05)^{3} = (1 + I)^{3} I = 0.05 ; I = i/2 = 0.05, i =0.10 or 10% p.a.

Illustration 9: Solving for Missing Variables

A certain sum invested at 4% per annum compounded semi- annually amounts to Rs. 78030 at the end of one year. Find the sum. Solution: Here A = 78030 m=2, N = mn = 2 x 1 = 2, I = i/m = i/2 = 0.04 / 2 = 0.02 We have, A = P(1 + I)^{N} A = P(1 + 0.02)^{2} 78030 = P (1.02)^{2} P = 78030 / 1.02^{2} = 75000 Thus Rs. 75,000 is to be invested at 4% p.a. compounded semi annually to get an amount of Rs. 78,030 at the end of 1 year.	Rs. 16000 invested at 10% p.a. Compounded Semi-Annually amounts to Rs. 18,522. Find the time period of investment. **Solution:** Here P = Rs. 16,000 , m = 2 , A = Rs. 18,522 m = 2 ,1 = i/m = 10 x $^{1}/_{2}$ % = 5% = 0.05 N =mn = 2n **(Required)** We have A = P(1 + I)^{N} 18,522 =16,000(1 + 0.05)^{N} 18,522/ 16,000= (1.05)^{N} (1.157625) = (1.05)^{N} (1.05)^{3} = (1.05)^{N} N = 3, N =2n = 3, **n** = **1.5 years.** ∴ Required no. of years = 1.5 years.

Illustration 10: Solving for Missing Variables

A person opened an account on April, 2017 with a deposit of Rs. 800. The account paid 6% interest compounded quarterly. On October 1 2017 he closed the account and added enough additional money to invest in a 6 months time-deposit for Rs.1000, earning 6% p.a. compounded monthly.

(a) How much additional amount did the person invest on October 1?

(b) What will be the maturity value of his deposit on April 1 2018?

(c) How much total interest is earned?

Given that $(1 + i)^n$ is 1.03022500 for $i = 1^1/_2$ %, n = 2 and $(1 + i)^n$ is 1.03037751 for $i = {}^1/_2$% and n = 6.

Solution:

(a) The Amount Received from the Initial Deposit of Rs. 800 for 6% Compounded Quarterly. **Given:** P = 800, m = 4, I = i/4 = 0.06 / 4 = 0.015, N=mn = 4 × 0.5 = 2, **Reqd:** Amount Earned at the End of the Closure Period of 6 Months - A. $A = P(1 + I)^N$ **A** = $800(1 + 0.015)^2 = 824.18$ ∴Additional Amount Invested to Reach the Required principal of Rs. 1,000 = 1,000 - 824.18 = 175.82	(b) Amount from Rs. 1,000 time deposit. **Given:** P = 1,000, m = 12, I = i/12 = 0.06 / 12 = 0.005, N=mn = 12 × 0.5 = 6, **Reqd:** Amount Earned at the End of the Closure Period of 6 Months - A. $A = P(1 + I)^N$ $A = 1{,}000\ (1 + 0.005)^6$ A = 1,000 (1.0304) A = 1,030.40 Total Interest Earned = Rs. 24.18 + 30.40 = Rs. 54.58

4.3 EFFECTIVE INTEREST KATE

Illustration 11: Effective Interest Rate

Ascertain the Effective Rate of Interest in the following situations -

1. The Total Interest Paid during the Period = Rs. 100, the Amount Received at the End of the Period = Rs. 1,100.

2. The Interest Rate is 10%p.a compounded annually, The Opening Principal = Rs. 100. Find the Effective Interest Rate.

Solution:

1. Effective Interest Rate = $\frac{\text{Total Interest during the year}}{\text{Opening Principal}}$

Effective Interest Rate = $\frac{100}{1{,}100 - 100} = \frac{100}{1{,}000}$ = 10% p.a.

2. (a) Total Interest During the Year under Compound Interest = $P[(1+i)^n - 1] = 100(1+0.10)^1]$ = Rs. 10

(b) Opening Principal = Rs. 100

(c) Effective Interest Rate = $\frac{\text{Total Interest during the year}}{\text{Opening Principal}} = \frac{10}{100}$ = 0.10 or 10% p.a.

Note: The Effective Interest Rate = Actual Interest Rate where the Interest is Compounded only once during a year.

Illustration 12: Effective Interest Rate

Rs. 5000 is invested in a Term Deposit Scheme that fetches interest 6% per annum compounded quarterly. What will be the interest after one year? What is the Effective Rate of Interest?

Solution:

We know that under Compound Interest, the Total Interest during a given period is –

Interest = P $[(1 + I)^N - 1]$

P = Rs. 5000 m = 4 I = i/m = 0.06 / 4 = 0.15 N = mn = 4n = 4×1 = 4

Interest = Rs. 5000 $[(1 + 0.015)^4 - 1]$

Rs. 5000 × 0.06136355

= Rs. 306.82

Effective Interest Rate = $\frac{\text{Total Interest during the year}}{\text{Opening Principal}} = \frac{306.82}{5,000}$ = 0.061364 = 6.1364% p.a.

Alternatively, Effective Interest Rate = E = $(1 + i/m)^m - 1 = (1 + 0.06/4)^4 - 1$ = 1.06136 - 1 = 0.06136 = 6.136% p.a.

Illustration 13: Effective Interest Rate - Application

Answer the Following -

Find the amount of compound interest and effective rate of interest if an amount of Rs. 20,000 is deposited in a bank for one year at the rate of 8% per annum compounded semi annually. **Solution:** We know that under Compound Interest, Total Interest during a period = P $[(1 + I)^N - 1]$ P = Rs. 20,000 m = 2 I = i/m = i/ 2 = 0.08 / 2 = 0.04 N = mn = 2×1 = 2 I = Rs. 20,000 $[(1 + 0.04)^2 - 1]$ = Rs. 20,000 × 0.0816 = Rs. 1,632 Effective rate of interest(E)= $[(1 + i/m)^m - 1]$ = $[(1 + 0.04)^2 - 1]$ = 0.0816 = 8.16%	Which is a better investment 3% per year compounded monthly or 3.2% per year simple interest? Given that (1 + $0.0025)^{12}$ = 1.0304. **Solution:** 1. Under Simple Interest Effective Interest Rate = Actual Interest Rate = 3.2% [Since there is no question of Compounding in Simple Interest] 2. Under Compound Interest, Effective Interest Rate = E = $(1 + i/m)^m - 1$ m = 12 I = i/m = .03/12 = 0.0025 Effective Interest Rate = E = $(1 + i/rn)^m - 1$ = $(1 + 0.0025)^{12} - 1$ = 1.0304 - 1 = 0.0304 = 3.04% Since the Effective Interest Rate is higher under the

Also, the Actual Interest During One Year may also be Calculated as follows - The Actual Interest during One Year = Opening Principal × 1 × Effective Interest Rate= 20,000 × 8.16% ×1= Rs. **1,632**	Compound Interest Scheme, the same may be preferred. **Note:** Students may verify the answer by Comparing the Actual Interest paid during the First Year using Notional Principal.

4.4 TIME VALUE OF MONEY AND ANNUITY

Illustration 14: Future Value and Present Value of Money

Answer the Following Questions -

You invest Rs. 3,000 in a two year investment that pays you 12% per annum. Calculate the Future Value of the investment. Solution: Future Value of an investment is the Amount that will be received from an investment at the end of N years. Future Value of Investment = F = C.F. $(1 + i)^n$ C.F. = Cash Flow = Rs. 3,000 i = rate of interest = 0.12 n = time period = 2 F = Rs. $3,000(1 + 0.12)^2$ = Rs. 3,000 × 1.2544 = Rs. **3,763.20**	**Find the Future Value of an Annuity of** Rs. 500 **made annually for 7 years at interest rate of** 14% **compounded annually. Given that** $(1.14)^7 = 2.5023$. Solution: Here Annual Payment = P = Rs. 500 n = 7 ; i = 14% = 0.14 Future value of the annuity = P(7, 0.14) = $=\frac{p}{i}$ $[(1+ j)^n -1]$ $= \frac{500}{0.14} \times [(1 + 0.14)^7 -1] = \frac{500\times(2.5023-1)}{0.14}$ = Rs 5365.36
Rs. 200 is invested at the end of each of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10th payment? Given that $(1.005)^{10} = 1.0511$ Solution: Regular payment = P = Rs. 200 Total No. of Payment = N = 10 I = i/m = 0.06/12 = 0.005 P(n, i) = $\frac{P}{I}$ x $[(1 + I)^N - 1]$ P(10, 0.0050) = $\frac{200}{0.005}$ $[(1 + 0.005)^{10} - 1]$ $= \frac{200}{0.005}$ [(1.0511 - 1] - 40,000 x0.0511 **= Rs. 2,044**	Z invests Rs. 10,000 every year starting from today for next 10 years. Suppose interest rate is 8% per annum compounded annually, calculate Future Value of the Annuity. Given that $(1 + 0.018)^{10} = 2.15892500$. Solution: The Given problem is an "Annuity Immediate" problem, i.e the regular cash payments are made at the Beginning of the year. Step 1: p Future Value of the Annuity Immediate = $\frac{P}{I}$ $[(1 + i)^n - 1] (1+i)$ $= \frac{10000}{0.08}$ $[1 + 0.08)^{10} - 1]$ (1-08) = 1,25,000 × 1.1589 × 1.08 = Rs. **1,56,451.50**

Illustration 15: Future Value and Present Value of Money Answer the Following Questions -

<table>
<tr>
<td>What is the present value of Rs. 1 to be received after two years compounded annually at 10% interest rate?

Solution:

Present Value is the Amount to be invested now to get a specified amount at the end of the period.

Required Amount = A = Rs. 1

i = 10% = 0.1, n = 2

We know that, A = P$(1+i)^n$, therefore P = A / $(1+i)^n$

Required present value (P) =$\frac{A}{(1+i)^n)}$ =$\frac{1}{(1+0.1)^2}$

= 1/ 1.21= 0.8264 = Rs. 0.83

Conclusion: Rs. 0.83 is required to be invested now to obtain Rs. 1 after 2 years at 10% interest rate compounded annually.</td>
<td>Find the present value of Rs. 10,000 to be required after 5 years if the interest rate be 9%. Given that $(1.09)^5$ = 1.5386.

Solution:

Here i = 0.09

n = 5

A = 10,000

Required Present Value(P) $\frac{A}{(1+i)^n}$=$\frac{10000}{(1+0.09)^5}$

= $\frac{10000}{1.5386}$ Rs. 6,499.42</td>
</tr>
<tr>
<td colspan="2">What is the Present Value of an Annuity consisting of payments of Rs. 10,000 at 10% compounded annually, paid for 10 years?

Solution:

Step 1: Ascertain the Future Value of the Annuity

Future Value of An annuity = $\frac{P}{i}$ [$(1 + i)^n$ - 1] where P=Regular Payments=10,000, i=0.10, n = 10 years.

= $\frac{10,000}{0.1}$ [$(1 + 0.1)^{10}$] = 1,00,000 × 1.5937 = Rs. 1,59,370.

Step 2: Ascertain the Present Value of the Annuity

The Present Value of the Annuity and the Future Value of the Annuity are related through the following formula -

Present Value of the Annuity = $\frac{\text{Future Value of an annuity}}{(1+i)^n}$ = 1,59,370 / $(1+0.1)^{10}$ = Rs. 61,445.04</td>
</tr>
</table>

Illustration 16: Applications of Annuity Formula

S borrows Rs. 5,00,000 to buy a house. If he pays equal instalments for 20 years and 10% interest on outstanding balance what will be the Equal Annual Instalment? Solution:	Rs. 5,000 is paid every year for ten years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually? **Solution:**

Loan Amount = Rs. 500000 ; n = 20 ; i = 10% p.a. = 0.10 **Step 1:** Calculate the Future Value of the Loan Amount. F.V. = A = P $(1+i)^n$ = $5,00,000(1.1)^{20}$ = 5,00,000 (6.7275) = 33,63,750 **Step 2:** Calculate the Annual Payments equal to the Required Future Value of the Loan Future Value of An annuity = $\frac{P}{I}$ $[(1 + i)^n - 1]$ 33,63,750 = $\frac{P}{0.1}[(1 + 0.I)^{20}-1]$ 3,36,375 = P [5.7275] P = 3,36,375 / 5.7275 = Rs. **58,729.8123**	P = Rs. 5,000 n = 10 i = 0.14 **Step 1:** Calculate the Future Vaiue of the Loan Amount = F.V. of Annuity = $\frac{P}{i}$ $[(1 + i)^n - 1]$ =$\frac{5000}{0.14}$ $[(1 + 0.14)^{10} - 1]$ = 35,714.2857 × 2.7072 = **Rs. 96,685.7142** **Step 2:** Calculate the Present Value of the Loan Amount P = A / (1+i) = 96,685.7142 / $(1.14)^{10}$ = **26,080.5228** Hence the Loan amount presently disbursed is Rs. 26,080.5228, for a ten annual repayments of Rs. 5,000 at 14% p.a.
Y bought a TV costing Rs. 13,000 by making a down payment of Rs. 3,000 and agreeing to make equal annual payment for four years. How much would be each payment if the interest on unpaid amount be 14% compounded annually? **Solution:** Here the Loan Amount is Rs. 10,000 i.e. Rs. 13,000 - Rs. 3,000 and we have to calculate Equal Annual Payment over the period of Four Years. **Given:** Loan Amount = Rs. 10,000;n = 4; i=14% p.a. = 0.14 **Required:** Regular Payments (P) **Step 1:** Calculate the Future Value of the Loan Amount. F.V. = A = P $(1+i)^n$ = $10,000(1.14)^4$ = 10,000 (1.6890) = **Rs. 16,890** **Step 2:** Calculate the Annual Payments equal to the Required Future Value of the Loan F.V. Annuity = $\frac{P}{i}$ $[(1 + i)^n - 1]$ 16890 = $\frac{P}{0.14}$ $[(1+ 0.14)^4-1]$ 2364.60 = P [0.6890] P = 2364.60 / 0.6890 = Rs. **3,431.9303**	Suppose your mom decides to gift you Rs. 10,000 every year starting from today for the next five years. You deposit this amount in a bank as and when you receive and get 10% per annum interest rate compounded annually. What is the Present Value of this annuity? **Solution:** Since the Payments are starting today, it is an annuity immediate. **Step 1:** Calculate the F.V. of Annuity Immediate Future Value of the Annuity Immediate = $\frac{10,000}{0.10}$ $[(1 + 0.1)^5-1](1 + 0.1)$ = 1,00,000 [0.61051](1.1) = 67,156.10 **Step 2:** Present Value of the Annuity Immediate = F.V. of Annuity Immediate / $(1+i)^n$ Present Value of the Annuity Immediate = 67,156.10 / $(1.1)^5$ = 67,156.10 / 1.6105 = Rs. **41,698.9134**

Illustration 17: Leasing Decisions

ABC Ltd. wants to lease out an asset costing Rs. 3,60,000 for a five year period. It has fixed a rental of Rs. 1,05,000 per annum payable annually starting from the end of first year. Suppose rate of interest is 14% per annum compounded annually on which money can be invested by the company. Is this agreement favourable to the company? Given: P(5, 0.14) = 3.43308

Solution:

A. Present Value of the Rentai Income = Present Value of the Annuity of Rs. 1 @ 14% for 5 years × 1,05,000

= P(5,0.14) × 1,05,000

= 3.43308 × 1,05,000 = Rs. **3,60,473.40.**

B. Cost of the Asset = Rs. **3,60,000.**

C. Net Benefit = **473.40.**

D. Since the Net Benefit is positive the Asset may be leased out at the given parameters.

Note: The Present Value of an Annuity of Rs. 1 for n years can be obtained from the Table given in **Pg. No. T.2**

Illustration 18: Leasing Decisions

A company is considering proposal of purchasing a machine either by making full payment of Rs. 4000 or by leasing it for 4 years at an annual rental of Rs. 1250. Which course of action is preferable if the company can borrow at 14% Compounded Annually?

Solution:

1. The present value of an Annuity = Present Value of Rs. 1 for 4 years at 14% × 1250

= 1250 × P (4, 0.14)

= 1250 × 2.91371

= Rs. 3,642.14

∴ The Present Value of the Lease Payments = Rs. 3,642.14.

2. The Cost of the Machine if payment is made outright = Rs. **4,000.**

3. **Conclusion:** Since the Present Value of the Lease Payments is lower, the Company may to buy the Machine on Lease. The same will result in a savings of Rs. 357.86.

Illustration 19: Capital Investment Decisions

A machine can be purchased for Rs. 50,000. Machine will contribute Rs. 12,000 per year for the next five years. Assume borrowing cost.is 10% per annum compounded annually. Determine whether machine should be purchased or not.

Solution:

1. The Present Value of Annual Benefit = A × P (n, i)

= 12,000 x P(5, 0.10)

= 12,000 x 3.79079

= Rs. 45,489.48

2. Initial Cost = Rs. 50,000

3. Net Loss = Rs. 4,510.52. Hence the Machine must not be purchased.

Illustration 20: Effective Savings - Investment Decisions

A machine with useful life of seven years costs Rs. 10,000 while another machine with useful life of five years costs Rs. 8,000. The first machine saves labour expenses of Rs. 1,900 annually and the second saves labour expenses of Rs. 2,200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% compounded per annum.

Solution:

The present value of annual cost savings for the first machine = Rs. 1,900 x P (7, 0.10) = Rs. 1,900 x 4.86842 = Rs. 9249.99 = Rs. 9250 Cost of machine being Rs. 10,000 it costs more by Rs. 750 than it saves in terms of labour cost.	The present value of annual cost savings of the second machine = Rs. 2,200 x P(5, 0.10) = Rs. 2,200 x 3.79079 = Rs. 8,339.74 Cost of the second machine being Rs. 8000 effective savings in labour cost is Rs. 339.74. Hence the second machine is preferable.

Illustration 21: Bond Valuation

An investor intends purchasing a three year Rs. 1,000 par value bond having nominal interest rate of 10%. At which price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

Solution:

Present values of the bond $= \frac{100}{(1+0.14)^1} + \frac{100}{(1+0.14)^2} + \frac{100}{(1+0.14)^3} + \frac{100}{(1+0.14)^3}$

= [100 × 0.87719] + [100 × 0.769467] + [100 × 0.674 972] + [1,000 × 0.674972]

= 87.719 + 76.947 + 67.497 + 674.972 = Rs. **907.125**

Thus the Purchase Value of the bond is Rs. 907.125. This represents the Maximum Value for which the Bond may be purchased.

Illustration 22: Bond Valuation

A is willing to purchase a five years Rs. 1,000 Par Value Bond having a Coupon Rate of 9%. A's Required Rate of Return is 10%. How much A should pay to purchase the Bond, if it matures at par?

Solution: Note: Discounting should be done at the Required Rate of Return, i.e. Desired Yield = 10% in this case.

Nature	Period	Cash Flow	DF at 10% for 5 Periods	DCF
Interest (1,000 × 9%)	1-5	90	PVIFA = 3.791	341.19
Maturity Amount	5	1,000	PVIF = 0.621	621.00
Intrinsic Value				**Rs. 962.19**

Illustration 23: Bond Valuation- Half yearly payment

A 6 years Bond of Rs. 1,000 has an annual rate of interest of 14 %. The interest is paid half yearly. If the Required Rate of Return is 16 %, what is the Value of the Bond?

Nature	Period	Cash Flow	DF at 8% for 12 Periods	DCF

Half Yearly Interest (1,000 × 14% × 6/12)	1-12	70	PVIFA = 7.536	527.52
Maturity Amount	12	1,000	PVIF = 0.397	397.00
Intrinsic Value				**Rs. 924.52**

Note: Since Interest is payable half yearly, Present Value at the end of 6th Year is to be computed based on the half—

yearly interest rate of 8%, and the number of periods as 12.

Illustration 24: Bond Valuation- Half yearly payment

Calculate Market Price of—

1. 10% Government of India Security currently quoted at Rs. 110, but Interest Rate is expected to go up by 1%.

2. A Bond with 7.5% Coupon Interest, Face Value Rs. 10,000 and term to maturity of 2 years, presently yielding 6% interest payable half-yearly.

Solution: 1. 10% Government of India Bonds:

(a) Current Yield = $\frac{\text{Interest Amount}}{\text{Current Market Price}} = \frac{10\% \times \text{Rs.l00}}{\text{Rs.110}} = \bullet \frac{\text{Rs.10}}{\text{Rs.110}}$ = 9.09%

Note: It is assumed that the Face Value is Rs. 100, and the Bond has a perpetual life,

(b) Revised Yield = 9.09% + 1% = 10.09%

(c) Revised Market Price = $\frac{\text{Interest Amount Rs.10}}{\text{Market Yield 10.09\%}}$ **= Rs. 99.10**

2. 7.5% Bond

Nature	Period	Cash Flow	DF at 3% for 4 Periods	DCF
Half Yearly Interest (10,000 × 7.50% × 6/12)	1-4	375	PVIFA = 3.717	1,393.88
Maturity Amount	4	10,000	PVIF = 0.888	8,880.00
Market Price				**Rs.10,273.88**

Illustration 25: Bond Valuation- Different Yield Rates

10% Government of India Bonds (Annual Interest Payment) have five years to maturity and a Maturity Value of Rs. 10,000. Ascertain the value of the Bond today if the desired yields on such Bonds are 8%, 10% and 12%. Assuming the Desired Yield is 8%, and presently the Bond is traded at Rs. 11,500, what would you do?

Solution:

Yr	Nature	CF	Yield @ 8%		Yield @ 10%		Yield @ 12%	
			PVF @ 8%	DCF	PVF @ 10%	DCF	PVF @ 12%	DCF
1-5	Interest (10% × 10,000)	1,000	3.993	3,993	3.791	3,791	3.605	3,605

5	Maturity Value	10,000	0.681	6,810	0.621	6,210	0.567	5,670
	Value of Bond today			**10,803**		**10,001**		**9,275**

Action on Bond: If the Bond is traded at Rs. 11,500 now, its Current Market Price is greater than (Theoretical Value = Expected Price) at 8%, i.e. Rs. 10,803. This means that the Bond is overpriced. Hence, the Investor should **SELL** the Bond.

Illustration 26: Compounded Annual Growth Rate

The following data relating to Investment made by a Company for the past 5 years.

Years	1	2	3	4	5
Closing Market Price (Rs.)	50.00	64.00	85.00	100.00	125.00
Dividend Yield (Rs.)	4.00	8.00	10.00	15.00	15.00

Opening Market Price in Year 1 was Rs. 40. Also ascertain the Compounded Annual Growth Rate. What would be the Annual Growth Rate if there were no Dividend Payouts at all?

Solution: Computation of Compounded Annual Growth Rate

CAGR = $\sqrt[n]{\frac{\text{Total Return + Initial Investment}}{\text{Initial Investment}}}$ -1 [Inverse of Compound Interest Formula]

OR = $[\frac{\text{Tota Return+Initial Investment}}{\text{Initia1 Investment}}]^{-1/n} - 1$where, "n" represents the period of holding.

(a) CAGR with Dividend Pavouts:	**(b) CAGR without Dividend Pavouts:**
$= \sqrt[n]{\frac{\text{Total Return+Initil Investment}}{\text{Initial Investment}}} - 1$ $= [\frac{137+40}{40}]^{1/5} - 1 = (4.425^{1/5} - 1)$ $= 1.3464 - 1 = 0.34640 \text{ or } 34.64\%$	$= \sqrt[n]{\frac{\text{Capital Appreciation Return+Initia Investment}}{\text{Initial Investment}}}$ $= [\frac{85+40}{40}]^{1/5} - 1 = (3.125^{1/5} - 1)$ $= 1.2559 - 1 = 0.25590 \text{ or } 25.59\%$

Illustration 27: Real vs Nominal Cash Flows - Discount Rate

Following are the expected Nominal Cash Flows of Project A, requiring an initial outlay of Rs. 1,00,000.

Year	1	2	3	4
Cash Flow	Rs. 30,000	Rs. 60,000	Rs. 40,000	Rs. 10,000

If the Cost of Capital of the Company in a static economy is 8%, and Inflation Rate is 5%, what is the appropriate Discount Rate for the Project? Should the Project be accepted?

Solution:

1.Determination of Discount Rate

Since cash-flows are expressed in Nominal Terms (after including the impact of inflation), the discount rate chosen should also be Nominal Discount Rate, i.e. Real Discount Rate adjusted for inflation.

Hence, $(1 + R_N) = (1 + R_R) \times (1 + I)$

Where, R_N = Nominal Discount Rate

R_R = Real Discount Rate (Without effects of inflation)

I = Inflation Rate

$(1 + R_N) = (1 + 8\%) \times (1 + 5\%)$

$= 1.08 \times 1.05 = 1.134$

$\mathbf{R_N} = 1.134 - 1 = 0.134$ or 13.4%

Therefore, appropriate Discount Rate is 13.40% for evaluating the project.

2. Evaluation of Project

Particulars	Year	Disc. Factor @ 13.4%	Cash Flow	DCF
Annual Operating Cash Inflow	1	0.882	30,000	26,460
	2	0.777	60,000	46,620
	3	0.686	40,000	27,440
	4	0.605	10,000	6,050
Total Present Value of Cash Inflows				**1,06,570**
Less: Initial Investment	0	1.000	1,00,000	(1,00,000)
Net Present Value				**6,570**

The Project may be accepted since Net Present Value is positive.

4.5 PERPETUITY

Illustration 28: PV of a Constant Perpetuity

Ramesh wants to retire and receive Rs. 3,000 a month. He wants to pass this monthly payment to future generations after his death. He can earn an interest of 8% compounded annually. How much will he need to set aside to achieve his perpetuity goal?

PV of a **Constant** Perpetuity $=\frac{C}{R}$

Where **C** = Rs. 3,000

R = 0.08/12 or 0.00667

Substituting these values in the above formula, we get PV = $\frac{3,000}{0.00667}$ **- Rs. 4,49,775**

Note: If Ramesh wants that the receipts of Rs. 3,000 should start from **today,** he must increase the size of the funds to handle the first payment, i.e. Rs. 4,52,775 which provides the immediate payment of Rs. 3,000 and leaves the balance Rs. 4,49,775 in the Fund to provide the future Rs. 3,000 payments.

Illustration 29: PV of a Growing Perpetuity

Assuming that the discount rate is 7% per annum, how much would you pay to receive Rs. 50, growing at 5%, annually forever?

Solution: PV of a **Growing** Perpetuity = $\frac{C}{R-G}$ Where **C** = Rs. 50, **R** = 0.07, and **G** = 0.05

Substituting these values in the above formula, we have PV = $\frac{50}{0.07-0.05}$ **=Rs. 2,500**

Illustration 30: Present Value of a Perpetuity

(a) A Company has issued 12% Preference Share Capital with a Face Value of Rs. 100. What would be Market Price of the Preference Shares if the rate of interest for the investor is 10%?

(b) A Company has paid Equity Dividend of Rs. 10 per share. Its profits and dividends are expected to grow at 5%. Calculate the Market Price of the Equity Shares, if the rate of interest for the investor is 24%

(a) Market Value of Preference Shares = PV of a **Constant** Perpetuity = $\frac{C}{R} = \frac{Rs.12}{10\%}$ **= Rs. 120**

Note: This means that the Investor will be ready to pay Rs. 120 to purchase a Preference Share and earn a dividend of 12% on its Face Value.

(b) Market Price of Equity Shares = PV of a **Growing** Perpetuity = $\frac{C}{R-G} = \frac{Rs.10}{24\%-5\%} = \frac{Rs.10}{19\%}$ **= Rs.52.63**